Square Difference Labeling Of Some Path, Fan and Gear Graphs

J.Shiama

Assistant Professor

Department of Mathematics

CMS College of Science and Commerce

Coimbatore -641049, Tamil Nadu, India

Email: jshiama@gmail.com

Abstract: In this paper it is proved that some new graphs admit square difference labeling. A function f is called a square difference labeling if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f^*:E(G) → N given by $f^*(u\ v) = |[f(u)]^2 - [f(v)]^2|$ for every uv ∈ E(G) are all distinct. Any graphs which satisfies the Square difference labeling is called the Square difference graph. Here it is investigated that the central graphs of path, square graphs of path, some path related graphs, fan and gear graphs are all square difference graphs.

Keywords: Square difference labeling**,** square difference graphs, central graphs and power graphs.

1. INTRODUTION

All graphs in this paper are finite and undirected. For all other terminology and notations I follow Harary [2] .Let G (V,E) be a graph where the symbols V (G) and E (G) denotes the vertex set and the edge set. The cardinality of the vertex set is called the order of the graph G and it is denoted by p. The cardinality of the edge set is called the size of the graph G and it is denoted by q. Hence the graph is denoted by G (p, q).If the vertices of the graph are assigned values subject to certain conditions is known as graph labeling. The definitions for power graphs are used from Gary Chatrand [4].Some basic concepts are taken from [5] and [11]

A dynamic survey on graph labeling is regularly updated by Gallian [3] and it is published by Electronic Journal of Combinatorics. The square sum labeling is previously defined by V.Ajitha, S.Arumugam and K.A.Germina [1]. I proved some middle and total graphs [6] are square sum graphs.

The concept of square difference labeling was first introduced by J.Shiama [9]. Myself proved in [9] and [10] many standard graphs like P_n , C_n , complete graphs , cycle cactus, ladder, lattice grids, quadrilateral snakes , Wheels, K_2+ m K_1 , comb , star graphs, m K_3 , m C_3,duplication of vertices by an edge to some star graphs and crown graphs are square difference graphs. Also proved that the path is an odd square difference graphs and star graphs are perfect square graphs. Some graphs like shadow and split graphs [7], [8] can also be investigated for the square difference labeling.

Result-1: Path graphs are square difference graphs [10]

Result-2: Wheels graphs are square difference graphs [9]

2. MAIN RESULTS

Definition2.1: Let G = (V(G),E(G)) be a graph .A function f is called a square difference labeling if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f^*:E(G) → N given by

$$f^*(u\,v) = |[f(u)]^2 - [f(v)]^2|$$

for every uv ∈ E(G) are all distinct.

Definition2.2: Any graph which admits square difference labeling is called square difference graph.

Definition2.3: The k^{th} power G^k of a connected graph G,where k≥1, is that graph with $V(G^k)$ = V(G) for which uv ∈ $E(G^k)$ if and only if $1 \le d_G(u,v) \le k$ .The graphs G^2 and G^3 are also referred to as square and cube respectively of G.

Definition2.4: Let G be a finite undirected graph with no loops and multiple edges. The central graph C(G) of a graph G is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G. By definition vertices of the central graph is $P_{c(G)}$ = p + q , where p and q are number of vertices and edges in G. For any graph (p,q) there exists p vertices of degree p-1 and q vertices of degree 2 in C(G)

Definition2.5: The graph (P_n; K_1) is obtained from a path P_n by joining a pendant edge at each vertex of the path.

Definition2.6: Let S_m be a star graph with vertices v, $w_1, w_2, \ldots, w_m$. Define (P_n; S_m) the graph obtained from m copies of S_m and the path P_n: $u_1, u_2, \ldots, u_n$ by joining u_j with the vertex v of the j^{th} copy of S_m by means of an edge ,$1 \le m \le 3$, $1 \le j \le n$.

Theorem- 2.7: Central graph of a path P_n admits square difference labeling

Proof: Let P_n : $u_1, u_2, \ldots, u_n$.

Sub divide the edges by $v_1, v_2, \ldots, v_{n-1}$. Let G = C(P_n)

p = | V (G) | = 2n-1 and q = | E (G) | = (n-1)(n+2)/2

Define f: V (G) →{0,1,2,….p-1} as follows

$f(u_i) = 2i-2,\ 1 \le i \le n-1$

$f(v_i) = 2i-1,\ 1 \le i \le n-1$

We construct the vertex labeled sets as follows.

$$V_1 = \bigcup_{i=1}^{n} \{ f\ (u_i) \}$$

$$= \bigcup_{i=1}^{n} \{2i-2\}$$

$$= \{0,2,4, , \ldots , 2n-2\}$$

$$V_2 = \bigcup_{i=1}^{n} \{ f\ (v_i) \}$$

$$= \bigcup_{i=1}^{n} \{2j-2\}$$

$$= \{1, 3, 5, \ldots , 2n-3\}$$

We construct the edge labeled sets as follows.

$$E_1 = \bigcup_{i=1}^{n-1} \{ f^*(u_i v_i) \}$$

$$= \bigcup_{i=1}^{n-1} \{ |(2i-2)^2 - (2i-1)^2| \}$$

ISSN 2229-5518

$$= \bigcup_{i=1}^{n-1} \{| -4i + 3 |\}$$

$$= \bigcup_{i=1}^{n-1} \{4i - 3\}$$

= {1,5,9, . . .,4n-7}

$$E_2 = \bigcup_{i=1}^{n-1} \{ f*(v_i u_{i+1})\}$$

$$= \bigcup_{i=1}^{n-1} \{| [f(v_i)]^2 - [f(u_{i+1})]^2 |\}$$

$$= \bigcup_{i=1}^{n-1} \{(| 2i-1)^2 - (2i)^2 |\}$$

$$= \bigcup_{i=1}^{n-1} \{| -4i + 1 |\}$$

$$= \bigcup_{i=1}^{n-1} \{4i - 1\}$$

= {3,7,11, . . .,4n-5}

$$E_3 = \bigcup_{i=1}^{n-1} \{ f*(u_i u_{i+2})\}$$

$$= \bigcup_{i=1}^{n-1} \{| [f(u_i)]^2 - [f(u_{i+2})]^2 |\}$$

$$= \bigcup_{i=1}^{n-1} \{(| 2i-2)^2 - (2i+2)^2 |\}$$

$$= \bigcup_{i=1}^{n-1} \{16i \}$$

={ 16,32, . . . }

$$E_4 = \bigcup_{i=1}^{n-1} \{ f*(u_i u_{i+3})\}$$

$$= \bigcup_{i=1}^{n-1} \{| [f(u_i)]^2 - [f(u_{i+3})]^2 |\}$$

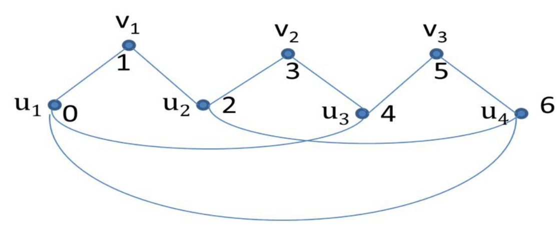
$$= \bigcup_{i=1}^{n-1} \{(| 2i-2)^2 - (2i+4)^2 |\}$$

$$= \bigcup_{i=1}^{n-1} \{| -24i - 12 \ |\}$$

= { 36, 60, . . . ,24n+12}

All the edges in all the sets are distinct. Hence the central graphs of path admit square difference labeling.

Example-2.8: Central graph of a path P_4 admits square difference labeling



3. Some path related graphs

Theorem- 3.1: The graph $P_n^{\ 2}$ is a square difference graph.

Proof: Let P_n: $u_1, u_2, \ldots, u_n$ be a path. Let G = $P_n^{\ 2}$

p = | V (G) | = n and q = | E (G) | = 2n-3.

Define a vertex labeling f: V (G) → {0,1,2, . . .p-1} by

$f(u_i) = i-1$, $1 \le i \le n$

and the induced edge labeling function

f^* : E (G) → N defined by

$f^*(u\ v) = |[f(u)]^2 - [f(v)]^2|$ for every uv ∈ E(G) are all distinct.

such that $f^*(e_i) \neq f^*(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_1 = \{(u_iu_{i+1}) / 1 \le i \le n-1\}$

$E_2 = \{(u_iu_{i+2}) / 1 \le i \le n-2\}$

In E_1

$$f^*(u_iu_{i+1}) = \bigcup_{i=1}^{n-1} \{|[f(u_i)]^2 - [f(u_{i+1})]^2|\}$$

$$= \bigcup_{i=1}^{n-1} \{(|i-1)^2 - i^2|\}$$

$$= \bigcup_{i=1}^{n-1} \{|2i-1|\}$$

$=\{1,3,5, . . . 2n-3\}$

In E_2

$$f^*(u_iu_{i+2}) = \bigcup_{i=1}^{n-2} \{|[f(u_i)]^2 - [f(u_{i+2})]^2|\}$$

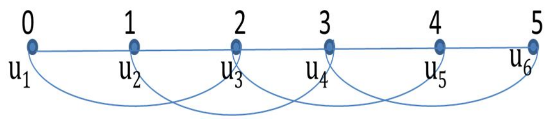
$$= \bigcup_{i=1}^{n-2} \{|(i-1)^2 - (i+1)^2|\}$$

$$= \bigcup_{i=1}^{n-2} \{|4i|\}$$

$=\{4,8,12, . . . 4(n-2)\}$

Clearly the labeling of edges of E_1 and that of E_2 are all distinct, as E_1 contains odd numbers and E_2 contains even numbers.Therefore $P_n{}^2$ are square difference graphs.

Example – 3.2: The graph $P_6{}^2$ is a square difference graph



Theorem- 3.3: The graph (P_n, K_1) is a square difference graph

Proof: Let $G = (P_n, K_1)$, $|V(G)| = 2n$ and $|E(G)| = 2n-1$.

Define the vertex labeling

$f: V(G) \to \{0,1,2,\ldots.p-1\}$ by

$f(u_i) = i-1,\ 1 \le i \le n$

$f(v_i) = i + n- 1,\ 1 \le i \le n$

and the induced edge labeling function

$f^*: E(G) \to N$ defined by

$f^*(uv) = |[f(u)]^2 - [f(v)]^2|$ for every $uv \in E(G)$ are all distinct.

such that $f^*(e_i) \neq f^*(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_1 = \{(u_iu_{i+1}) / 1 \le i \le n-1\}$

$E_2 = \{(u_iv_i) / 1 \le i \le n\}$

In E_1

$$f^*(u_iu_{i+1}) = \bigcup_{i=1}^{n-1} \{|[f(u_i)]^2 - [f(u_{i+1})]^2|\}$$

$$= \bigcup_{i=1}^{n-1} \{(|(i-1)^2 - i^2|\}$$

$$= \bigcup_{i=1}^{n-1} \{(|(2i-1)|\}$$

$=\{1,3,5, . . . 2n-3\}$

In E_2

$$f^*(u_iv_i) = \bigcup_{i=1}^{n} \{|[f(u_i)]^2 - [f(v_i)]^2|\}$$

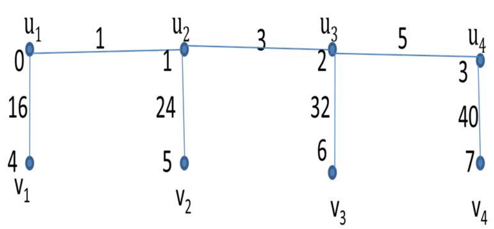
$$= \bigcup_{i=1}^{n} \{(|(i-1)^2 - (i+n-1)^2|\}$$

$$= \bigcup_{i=1}^{n} \{(|\,(2(i-1)n \;\; + n^2\ |\}$$

$=\{n^2, 2n+n^2, 4n+n^2, \ldots 3n^2 - 2n\}$

Clearly the edge labels are distinct. Hence the graphs (P_n, K_1) admit the square difference labeling.

Example – 3.4: The graph (P_n, K_1) is a square difference graph



Theorem- 3.5: The graph (P_n, S_1) is a square difference graph

Proof: Let G = (P_n, S_1) , | V (G) | = 3n and | E (G) | = 3n-1.

Define the vertex labeling

f: V (G) →{0,1,2,….p-1} by

$f(u_i) = i-1$, $1 \le i \le n$

$f(v_i) = i + n - 1$, $1 \le i \le n$

$f(w_i) = i + 2n - 1$, $1 \le i \le n$

and the induced edge labeling function

f^* : E (G) → N defined by

$f^*(u\ v) = |[f(u)]^2 - [f(v)]^2|$ for every uv ∈ E(G) are all distinct.

such that $f^*(e_i) \neq f^*(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_1 = \{(u_i u_{i+1}) / 1 \le i \le n-1\}$

$E_2 = \{(u_i v_i) / 1 \le i \le n\}$

$E_3 = \{(v_i w_i) / 1 \le i \le n\}$

In E_1

$$f^*(u_i u_{i+1}) = \bigcup_{i=1}^{n-1} \{|\,[f(u_i)]^2 - [f(u_{i+1})]^2\ |\}$$

$$= \bigcup_{i=1}^{n-1} \{(|\,i-1)^2 - (i)^2\ |\}$$

$$= \bigcup_{i=1}^{n-1} \{(|\,2i-1) \;\; |\}$$

$=\{1,3,5, \ldots 2n-3\}$

In E_2

$$f^*(u_i v_i) = \bigcup_{i=1}^{n} \{|\,[f(u_i)]^2 - [f(v_i)]^2\ |\}$$

$$= \bigcup_{i=1}^{n} \{(|\,(i-1)^2 - (n-1+i)^2\ |\}$$

$$= \bigcup_{i=1}^{n} \{(|\,(n^2 - 2n + 2ni\,|\}$$

$=\{n^2, 2n+n^2, 4n+n^2, \ldots 3n^2 - 2n\}$

In E_3

$$f^*(v_i w_i) = \bigcup_{i=1}^{n} \{|\,[f(v_i)]^2 - [f(w_i)]^2\ |\}$$

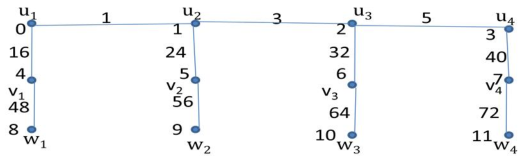
$$= \bigcup_{i=1}^{n} \{|\,(n-1+i)^2 - (2n-1+i)^2\ |\}$$

$$= \bigcup_{i=1}^{n} \{(|\,(-3n^2 + 2n - 2ni\,|\}$$

$=\{3n^2, 2n+3n^2, 4n+3n^2, \ldots 5n^2 - 2n\}$

Clearly the edge labels are distinct. Hence the graphs (P_n, S_1) are square difference graphs.

Example – 3.6: The graph (P_4, S_1) is a square difference graph



Theorem- 3.7: The graph (P_n, S_2) is a square difference graph

Proof: Let G = (P_n, S_2) , | V (G) | = 4n and | E (G) | = 4n-1.

Define the vertex labeling

f: V (G) →{0,1,2,….p-1} by

$f(u_i) = i-1,\ 1 \le i \le n$

$f(v_i) = n - 1 + i\ ,\ 1 \le i \le n$

$f(w_i) = 2n - 1 + i,\ 1 \le i \le 2n$

and the induced edge labeling function

$f^* :\ E(G) \to N$ defined by

$f^*(u\,v) = |[f(u)]^2 - [f(v)]^2|$ for every uv ∈ E(G) are all distinct.

such that $f^*(e_i) \ne f^*(e_j)$ for every $e_i \ne e_j$

The edge sets are

$E_1 = \{(u_iu_{i+1}) \,/\, 1 \le i \le n-1\}$

$E_2 = \{(u_iv_i) \,/\, 1 \le i \le n\}$

$E_3 = \{(v_iw_{2i-1}) \,/\, 1 \le i \le n\}$

$E_4 = \{(v_iw_{2i}) \,/\, 1 \le i \le n\}$

In E_1

$$f^*(u_iu_{i+1}) = \bigcup_{i=1}^{n-1}\{|[f(u_i)]^2 - [f(u_{i+1})]^2 |\}$$

$$= \bigcup_{i=1}^{n-1}\{(|(i-1)^2 - i^2 |\}$$

$$= \bigcup_{i=1}^{n-1}\{(|2i-1 \ |\}$$

={1,3,5, . . . 2n-3}

In E_2

$$f^*(u_iv_i) = \bigcup_{i=1}^{n}\{|[f(u_i)]^2 - [f(v_i)]^2 |\}$$

$$= \bigcup_{i=1}^{n}\{(|(i-1)^2 - (n-1+i)^2 |\}$$

$$= \bigcup_{i=1}^{n}\{(|(n^2 - 2n + 2ni |\}$$

$=\{n^2, 2n+n^2, 4n+n^2, \ldots 3n^2 -2n\}$

In E_3

$$f^*(v_iw_{2i-1}) = \bigcup_{i=1}^{n}\{|[f(v_i)]^2 - [f(w_{2i-1})]^2 |\}$$

$$= \bigcup_{i=1}^{n}\{|(n-1+i)^2 - (2n-2+2i)^2 |\}$$

$$= \bigcup_{i=1}^{n}\{(|3(n \ -1+i |\}$$

$= \{3n^2 , 3(n+1)^2, \ldots, 3(2n-1)^2 \}$

In E_4

$$f^*(v_iw_{2i}) = \bigcup_{i=1}^{n}\{|[f(v_i)]^2 - [f(w_{2i})]^2 |\}$$

$$= \bigcup_{i=1}^{n}\{|(n-1+i)^2 - (2n-1+2i)^2 |\}$$

$$= \bigcup_{i=1}^{n}\{(|(-3n^2 - 6ni - 3i^2 + 2i + 2n |\}$$

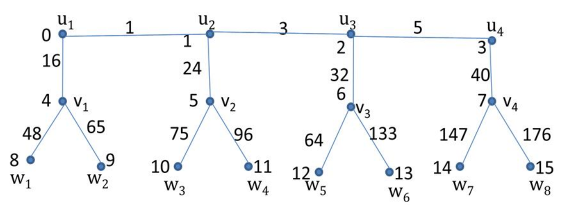
ISSN 2229-5518

$$= \bigcup_{i=1}^{n} \{(|3(n+i)^2 - 2(n+i|\}$$

All the edge labels in E_1, E_2, E_3 and E_4

are all distinct. Hence the graphs (P_n, S_2) are square difference graphs.

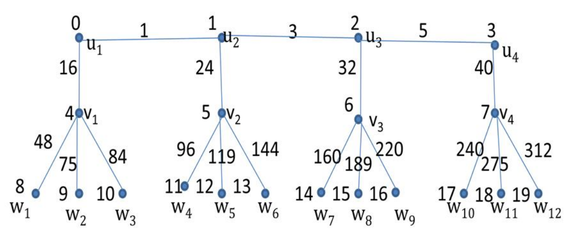
Example – 3.8: The graph (P_4, S_2) is a square difference graph



Theorem- 3.9: The graph (P_n, S_3) is a square difference graph

Proof9

Example – 3.10: The graph (P_4, S_3) is a square difference graph



4. Some other graphs

Theorem- 4.1: Fan graphs are square difference graphs.

Proof: Let F_n be a fan with vertices $u, u_1, u_2, \ldots, u_n$. Let $G = F_n$, | V (G) | = 2n+1 and

| E (G) | = 3n. Define the vertex labeling

f: V (G) →{0,1,2,….p-1} by

$f(u_i) = i$, $1 \le i \le n$

and f(c) = 0

and the induced edge labeling function

f^* : E (G) → N by

$f^*(u\,v) = |[f(u)]^2 - [f(v)]^2|$ for every uv ∈ E(G) are all distinct.

The edge sets are

$E_1 = \{(u_i u_{i+1}) / 1 \le i \le n-1\}$

$E_2 = \{(cu_i) / 1 \le i \le n\}$

The vertex sets are $V_1 = f(c) = 0$

$$V_2 = \bigcup_{i=1}^{n-1} f(u_i)$$

$$= \bigcup_{i=1}^{n-1} (i)$$

= {1,2, . . . ,n-1}

In E_1

$$E_1 = \bigcup_{i=1}^{n} \{ f^*(cu_i)\}$$

$$= \bigcup_{i=1}^{n} \{|[f(c)]^2 - [f(u_i)]^2|\}$$

$$= \bigcup_{i=1}^{n} |0 - (i)^2|\}$$

$$= \bigcup_{i=1}^{n} (i)_|^2$$

= {1,9,25, . . . ,(2n-1)²}

In E_2

$$E_2 = \bigcup_{i=1,3}^{n-1} \{ f^*(u_i u_{i+1})\}$$

$$= \bigcup_{i=,1,3}^{n-1} \{|[f(u_i)]^2 - [f(u_{i+1})]^2|\}$$

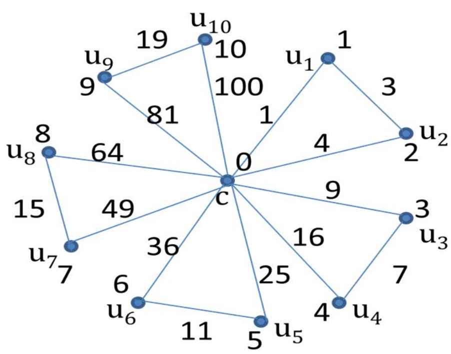
$$= \bigcup_{i=1,3}^{n-1}\{|(i)^2-(i+1)^2|\}$$

$$= \bigcup_{i=1,3}^{n-1}\{|2i+1|\}$$

={3,7,11, . . .,4n-1}

clearly the edge labels are distinct. Hence the Fans are all square difference graphs.

Example – 4.2: The fan graph F_5 is a square difference graph



Theorem- 4.3: Gear graphs G_n are square difference graphs.

Proof: Let | V (G_n) | = 2n+1 and | E (G_n) | = 3n.

Let us define the vertex labeling

f: V (G) → {0,1,2,….p-1} as follows

f(u) = 0

$f(u_i) = i$, $1 \le i \le n$

$f(v_i) = n + i$, $1 \le i \le n$

And the induced edge labeling function

f^* : E (G) → N defined by

$f^*(u\ v) = |[f(u)]^2 - [f(v)]^2|$ for every uv ∈ E(G) are all distinct.

such that $f^*(e_i) \neq f^*(e_j)$ for every $e_i \neq e_j$

The edge sets are

$E_1 = \{(u_i v_i) / 1 \le i \le n\}$

$E_2 = \{(v_i u_{i+1}) / 1 \le i \le n\}$

$E_3 = \{(u u_i) / 1 \le i \le n\}$

In E_1

$$f^*(u_i v_i) = \bigcup_{i=1}^{n}\{|[f(u_i)]^2-[f(v_i)]^2|\}$$

$$= \bigcup_{i=1}^{n}\{(|i)^2-(n+i)^2|\}$$

$$= \bigcup_{i=1}^{n}\{(|(n^2+2ni|\}$$

={48,60,72, . . . $3n^2$ }

In E_2

$$f^*(v_i u_{i+1}) = \bigcup_{i=1}^{n-1}\{|[f(v_i)]^2-[f(u_{i+1})]^2|\}$$

$$= \bigcup_{i=1}^{n-1}\{(|n+i)^2-(i+1)^2|\}$$

$$= \bigcup_{i=1}^{n-1}\{(|n^2+2ni-2i-1)\ \ |\}$$

= {45, 55, . . . }

$$E_3 = \bigcup_{i=1}^{n}\{f^*(u\ u_i)\}$$

$$= \bigcup_{i=1}^{n}\{|[f(u\ )]^2-[f(u_i)]^2|\}$$

$$= \bigcup_{i=1}^{n}\{|(0)^2-(i)^2|\}$$

$$= \bigcup_{i=1,}^{n}\{|i^2|\}$$

$=\{1,4,,\ldots,n^2\}$

In E_4

$$E_4 = \bigcup_{i=1}^{n} \{ f^*(u_1 v_n) \}$$

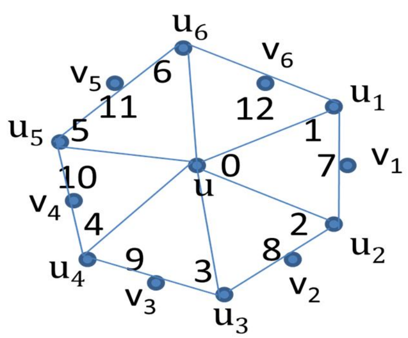
$$= \bigcup_{i=1}^{n} \{ | [f(u_1)]^2 - [f(v_n)]^2 | \}$$

$$= \bigcup_{i=1}^{n} \{ |1 - (2n)^2 | \}$$

$=\{4n^2 -1\}$

All the edges are distinct. Hence all the gear graphs G_n are square difference graphs.

Example – 4.4: The gear graph G_6 is a square difference graph



Conclusion: In this paper it is proved that some path graphs admits square difference labeling. Also proved some fan and gear graphs admit square difference labeling. In my previous papers [7],[8] it is proved that some graphs are square difference graphs. The square difference labeling can be investigated for more graphs. It is an open problem.

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